**Zadatak 161 (Matko5, gimnazija)**

Riješite sustav: $\begin{cases} x+y=2 \\ x\cdot y-z^2=1 \end{cases}.$

**Rješenje 161**

Ponovimo!

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad a^2=0 \Leftrightarrow a=0.$$

$$a^2+b^2=0 \Leftrightarrow a=b=0.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left.\begin{array}{l} x+y=2 \\ x\cdot y-z^2=1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} y=2-x \\ x\cdot y-z^2=1 \end{array}\right\} \Rightarrow \left[\begin{array}{c} \text{metoda} \\ \text{zamjene} \end{array}\right] \Rightarrow x\cdot(2-x)-z^2=1 \Rightarrow$$

$$\Rightarrow 2\cdot x-x^2-z^2=1 \Rightarrow 2\cdot x-x^2-z^2-1=0 \Rightarrow -x^2+2\cdot x-1-z^2=0 \Rightarrow$$

$$\Rightarrow -x^2+2\cdot x-1-z^2=0 \;/\cdot(-1) \Rightarrow x^2-2\cdot x+1+z^2=0 \Rightarrow (x-1)^2+z^2=0 \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x-1=0 \\ z=0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x=1 \\ z=0 \end{array}\right\}.$$

Računamo y.

$$\left.\begin{array}{l} x=1 \\ y=2-x \end{array}\right\} \Rightarrow y=2-1 \Rightarrow y=1.$$

Rješenje je uređena trojka:

$$(x, y, z)=(1, 1, 0).$$



**Vježba 161**

Odmor!

**Rezultat:** …

**Zadatak 162 (Alica, srednja škola)**

Riješite sustav: $\begin{cases} \frac{24}{x}-\frac{45}{y}=-2 \\ 9\cdot x+8\cdot y=2\cdot x\cdot y \end{cases}.$

**Rješenje 162**

Ponovimo!

$$\frac{a}{b}\cdot c=\frac{a\cdot c}{b} \quad , \quad \frac{a}{n}+\frac{b}{n}=\frac{a+b}{n} \quad , \quad \frac{b}{n}=\frac{b}{m} \Rightarrow n=m.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

1.inačica

$$\left.\begin{array}{l}\frac{24}{x}-\frac{45}{y}=-2\\ 9\cdot x+8\cdot y=2\cdot x\cdot y\end{array}\right\}\Rightarrow\left[\begin{array}{c}\text{uvjet}\\ x\neq 0\\ y\neq 0\end{array}\right]\Rightarrow\left.\begin{array}{l}\frac{24}{x}-\frac{45}{y}=-2\\ 9\cdot x+8\cdot y=2\cdot x\cdot y\ /\cdot\frac{1}{x\cdot y}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}\frac{24}{x}-\frac{45}{y}=-2\\ \frac{9\cdot x}{x\cdot y}+\frac{8\cdot y}{x\cdot y}=\frac{2\cdot x\cdot y}{x\cdot y}\end{array}\right\}\Rightarrow\left.\begin{array}{l}\frac{24}{x}-\frac{45}{y}=-2\\ \frac{9\cdot x}{x\cdot y}+\frac{8\cdot y}{x\cdot y}=\frac{2\cdot x\cdot y}{x\cdot y}\end{array}\right\}\Rightarrow\left.\begin{array}{l}\frac{24}{x}-\frac{45}{y}=-2\\ \frac{9}{y}+\frac{8}{x}=2\end{array}\right\}\Rightarrow\left.\begin{array}{l}\frac{24}{x}-\frac{45}{y}=-2\\ \frac{8}{x}+\frac{9}{y}=2\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left[\begin{array}{l}\text{metoda suprotnih}\\ \text{koeficijenata}\end{array}\right]\Rightarrow\left.\begin{array}{l}\frac{24}{x}-\frac{45}{y}=-2\\ \frac{8}{x}+\frac{9}{y}=2\ /\cdot 5\end{array}\right\}\Rightarrow\left.\begin{array}{l}\frac{24}{x}-\frac{45}{y}=-2\\ \frac{40}{x}+\frac{45}{y}=10\end{array}\right\}\Rightarrow\frac{24}{x}+\frac{40}{x}=-2+10\Rightarrow$$

$$\Rightarrow\frac{24+40}{x}=8\Rightarrow\frac{64}{x}=8\Rightarrow\frac{64}{x}=8\Rightarrow x=8.$$

Računamo y.

$$\left.\begin{array}{l}x=8\\ 9\cdot x+8\cdot y=2\cdot x\cdot y\end{array}\right\}\Rightarrow 9\cdot 8+8\cdot y=2\cdot 8\cdot y\Rightarrow 72+8\cdot y=16\cdot y\Rightarrow 8\cdot y-16\cdot y=-72\Rightarrow$$

$$\Rightarrow -8\cdot y=-72\Rightarrow -8\cdot y=-72\ /:(-8)\Rightarrow y=9.$$

Rješenje je uređeni par

$$(x,\ y)=(8,\ 9).$$



2.inačica

$$\left.\begin{array}{l}\frac{24}{x}-\frac{45}{y}=-2\\ 9\cdot x+8\cdot y=2\cdot x\cdot y\end{array}\right\}\Rightarrow\left[\begin{array}{c}\text{uvjet}\\ x\neq 0\\ y\neq 0\end{array}\right]\Rightarrow\left.\begin{array}{l}\frac{24}{x}-\frac{45}{y}=-2\\ 9\cdot x+8\cdot y=2\cdot x\cdot y\ /\cdot\frac{1}{x\cdot y}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}\frac{24}{x}-\frac{45}{y}=-2\\ \frac{9\cdot x}{x\cdot y}+\frac{8\cdot y}{x\cdot y}=\frac{2\cdot x\cdot y}{x\cdot y}\end{array}\right\}\Rightarrow\left.\begin{array}{l}\frac{24}{x}-\frac{45}{y}=-2\\ \frac{9\cdot x}{x\cdot y}+\frac{8\cdot y}{x\cdot y}=\frac{2\cdot x\cdot y}{x\cdot y}\end{array}\right\}\Rightarrow\left.\begin{array}{l}\frac{24}{x}-\frac{45}{y}=-2\\ \frac{9}{y}+\frac{8}{x}=2\end{array}\right\}\Rightarrow\left.\begin{array}{l}\frac{24}{x}-\frac{45}{y}=-2\\ \frac{8}{x}+\frac{9}{y}=2\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left[\begin{array}{c}\text{zamjena}\\ u=\frac{1}{x}\ ,\ v=\frac{1}{y}\end{array}\right]\Rightarrow\left.\begin{array}{l}24\cdot u-45\cdot v=-2\\ 8\cdot u+9\cdot v=2\end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{metoda suprotnih}\\ \text{koeficijenata}\end{array}\right]\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}24\cdot u-45\cdot v=-2\\ 8\cdot u+9\cdot v=2\ /\cdot 5\end{array}\right\}\Rightarrow\left.\begin{array}{l}24\cdot u-45\cdot v=-2\\ 40\cdot u+45\cdot v=10\end{array}\right\}\Rightarrow 64\cdot u=8\Rightarrow 64\cdot u=8\ /:64\Rightarrow u=\frac{8}{64}\Rightarrow$$

$$\Rightarrow u=\frac{8}{64}\Rightarrow u=\frac{1}{8}.$$

Računamo v.

$$\left.\begin{array}{l} u=\frac{1}{8} \\ 40\cdot u+45\cdot v=10 \end{array}\right\} \Rightarrow 40\cdot\frac{1}{8}+45\cdot v=10 \Rightarrow 40\cdot\frac{1}{8}+45\cdot v=10 \Rightarrow 5+45\cdot v=10 \Rightarrow$$

$$\Rightarrow 45\cdot v=10-5 \Rightarrow 45\cdot v=5 \Rightarrow 45\cdot v=5\ /:45 \Rightarrow v=\frac{5}{45} \Rightarrow v=\frac{5}{45} \Rightarrow v=\frac{1}{9}.$$

Sada računamo x i y.

$$\left.\begin{array}{l} \frac{1}{x}=u \\ \frac{1}{y}=v \end{array}\right\} \Rightarrow \left[\begin{array}{l} u=\frac{1}{8} \\ v=\frac{1}{9} \end{array}\right] \Rightarrow \left.\begin{array}{l} \frac{1}{x}=\frac{1}{8} \\ \frac{1}{y}=\frac{1}{9} \end{array}\right\} \Rightarrow \left.\begin{array}{l} x=8 \\ y=9 \end{array}\right\} \Rightarrow (x,\ y)=(8,\ 9).$$

**Vježba 162**

Odmor!

**Rezultat:** …

**Zadatak 163 (Martin, maturant)**

Za koju vrijednost parametra m sustav $\begin{cases} m\cdot x+3\cdot y=1 \\ 2\cdot x-\sqrt{3}\cdot y=7 \end{cases}$ nema rješenja?

$A.\ m=2\cdot\sqrt{3}$ $\quad B.\ m=\frac{\sqrt{3}}{2}$ $\quad C.\ m=-2\cdot\sqrt{3}$ $\quad D.\ m=-\frac{\sqrt{3}}{2}$



**Rješenje 163**

Ponovimo!

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad \left(\sqrt{a}\right)^2=a.$$

Parametar

Vladimir Anić, Ivo Goldstein, Rječnik stranih riječi, Novi Liber, Zagreb, 2002.

Veličina, obično realna varijabla, čije vrijednosti služe za razlikovanje elemenata nekog skupa točaka funkcija, jednadžbi ili drugih matematičkih objekata.

Bratoljub Klaić, Rječnik stranih riječi, Nakladni zavod MH, Zagreb, 1983.

Veličina o kojoj ovisi funkcija ili oblik krivulje.

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n} \ ,\ n\neq 0 \ ,\ n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ ,\ n\neq 0 \ ,\ n\neq 1.$$

Jednadžba pravca oblika

$$A\cdot x+B\cdot y+C=0$$

naziva se implicitni oblik jednadžbe pravca ili kraće, opći oblik jednadžbe pravca.

Jednadžba pravca oblika

$$y=k\cdot x+l$$

naziva se eksplicitni oblik jednadžbe pravca ili kraće, eksplicitna jednadžba pravca. Broj k naziva se

koeficijent smjera pravca. Broj l nazivamo odsječak pravca na osi y.
Uvjet usporednosti (paralelnosti):
Ako su pravci dani eksplicitnim jednadžbama $y = k_1 \cdot x + l_1$, $y = k_2 \cdot x + l_2$, tada su usporedni ako i samo ako je

$$k_1 = k_2.$$

**Jednadžba a · x = b nema rješenja, ako je a = 0, b ≠ 0.**
Sustav

$$\begin{cases} a_1 \cdot x + b_1 \cdot y = c_1 \\ a_2 \cdot x + b_2 \cdot y = c_2 \end{cases}$$

je nemoguć, tj. nema rješenja ako je

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}.$$

1.inačica

$$\left.\begin{array}{l} m \cdot x + 3 \cdot y = 1 \\ 2 \cdot x - \sqrt{3} \cdot y = 7 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow \left.\begin{array}{l} m \cdot x + 3 \cdot y = 1 \;/\cdot \sqrt{3} \\ 2 \cdot x - \sqrt{3} \cdot y = 7 \;/\cdot 3 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} m \cdot \sqrt{3} \cdot x + 3 \cdot \sqrt{3} \cdot y = \sqrt{3} \\ 6 \cdot x - 3 \cdot \sqrt{3} \cdot y = 21 \end{array}\right\} \Rightarrow m \cdot \sqrt{3} \cdot x + 6 \cdot x = 21 + \sqrt{3} \Rightarrow \left(m \cdot \sqrt{3} + 6\right) \cdot x = 21 + \sqrt{3}.$$

Budući da jednadžba a · x = b nema rješenja ako je a = 0, b ≠ 0, slijedi:

$$m \cdot \sqrt{3} + 6 = 0 \Rightarrow m \cdot \sqrt{3} = -6 \Rightarrow m \cdot \sqrt{3} = -6 \;/\cdot \frac{1}{\sqrt{3}} \Rightarrow m = -\frac{6}{\sqrt{3}} \Rightarrow m = -\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow$$

$$\Rightarrow m = -\frac{6 \cdot \sqrt{3}}{\left(\sqrt{3}\right)^2} \Rightarrow m = -\frac{6 \cdot \sqrt{3}}{3} \Rightarrow m = -\frac{6 \cdot \sqrt{3}}{3} \Rightarrow m = -2 \cdot \sqrt{3}.$$



Odgovor je pod C.

2.inačica

Svaka jednadžba s dvije nepoznanice predstavlja jednadžbu pravca. Grafički riješiti sustav dvije jednadžbe s dvije nepoznanice znači nacrtati za svaku jednadžbu odgovarajući pravac i naći sjecište tih pravaca. Rješenje sustava predstavlja koordinate presjeka tih pravaca. Ako su pravci usporedni sustav nema rješenja.
Napišemo eksplicitne oblike jednadžbi pravaca i odredimo koeficijente smjerova.

$$\left.\begin{array}{l} m \cdot x + 3 \cdot y = 1 \\ 2 \cdot x - \sqrt{3} \cdot y = 7 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 3 \cdot y = -m \cdot x + 1 \\ -\sqrt{3} \cdot y = -2 \cdot x + 7 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 3 \cdot y = -m \cdot x + 1 \;/\cdot \frac{1}{3} \\ -\sqrt{3} \cdot y = -2 \cdot x + 7 \;/\cdot \left(-\frac{1}{\sqrt{3}}\right) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} y = -\frac{m}{3} \cdot x + \frac{1}{3} \\ y = \frac{2}{\sqrt{3}} \cdot x - \frac{7}{\sqrt{3}} \end{array}\right\} \Rightarrow \left.\begin{array}{l} k_1 = -\frac{m}{3} \\ k_2 = \frac{2}{\sqrt{3}} \end{array}\right\}.$$

Pravci moraju biti paralelni jer sustav nema rješenja. To je moguće ako su koeficijenti smjera jednaki.

$$k_1 = k_2 \Rightarrow -\frac{m}{3} = \frac{2}{\sqrt{3}} \Rightarrow -\frac{m}{3} = \frac{2}{\sqrt{3}} \; / \cdot (-3) \Rightarrow m = -\frac{6}{\sqrt{3}} \Rightarrow m = -\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow$$

$$\Rightarrow m = -\frac{6 \cdot \sqrt{3}}{\left(\sqrt{3}\right)^2} \Rightarrow m = -\frac{6 \cdot \sqrt{3}}{3} \Rightarrow m = -\frac{6 \cdot \sqrt{3}}{3} \Rightarrow m = -2 \cdot \sqrt{3}.$$

Odgovor je pod C.

3.inačica

$$\left.\begin{array}{l} m \cdot x + 3 \cdot y = 1 \\ 2 \cdot x - \sqrt{3} \cdot y = 7 \end{array}\right\} \Rightarrow \left[\begin{array}{l} a_1 \cdot x + b_1 \cdot y = c_1 \\ a_2 \cdot x + b_2 \cdot y = c_2 \end{array}\right] \Rightarrow \left.\begin{array}{l} a_1 = m \; , \; b_1 = 3 \; , \; c_1 = 1 \\ a_2 = 2 \; , \; b_2 = -\sqrt{3} \; , \; c_2 = 7 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left[\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\right] \Rightarrow \frac{m}{2} = \frac{3}{-\sqrt{3}} \neq \frac{1}{7} \Rightarrow \frac{m}{2} = -\frac{3}{\sqrt{3}} \Rightarrow \frac{m}{2} = -\frac{3}{\sqrt{3}} \; / \cdot 2 \Rightarrow$$

$$\Rightarrow m = -\frac{6}{\sqrt{3}} \Rightarrow m = -\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow$$

$$\Rightarrow m = -\frac{6 \cdot \sqrt{3}}{\left(\sqrt{3}\right)^2} \Rightarrow m = -\frac{6 \cdot \sqrt{3}}{3} \Rightarrow m = -\frac{6 \cdot \sqrt{3}}{3} \Rightarrow m = -2 \cdot \sqrt{3}.$$

Odgovor je pod C.

**Vježba 163**

Odmor!

**Rezultat:** …



**Zadatak 164 (Robert, gimnazija)**

Riješite sustav $\left\{\begin{array}{l} x \cdot y + x + y = 1 \\ y \cdot z + y + z = 5 \\ x \cdot z + x + z = 2 \end{array}\right.$.

**Rješenje 164**

Ponovimo!

$$a = b \; , \; c \in R \Rightarrow a + c = b + c \quad , \quad a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad \sqrt{a^2} = a \; , \; a \geq 0.$$

$$\left.\begin{array}{l} a = b \\ c = d \end{array}\right\} \Rightarrow a \cdot c = b \cdot d \quad , \quad a^n \cdot b^n = (a \cdot b)^n .$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

$$\left.\begin{array}{l} x \cdot y + x + y = 1 \\ y \cdot z + y + z = 5 \\ x \cdot z + x + z = 2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x \cdot y + x + y = 1 \; / + 1 \\ y \cdot z + y + z = 5 \; / + 1 \\ x \cdot z + x + z = 2 \; / + 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x \cdot y + x + y + 1 = 2 \\ y \cdot z + y + z + 1 = 6 \\ x \cdot z + x + z + 1 = 3 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} (x \cdot y + x) + (y + 1) = 2 \\ (y \cdot z + y) + (z + 1) = 6 \\ (x \cdot z + x) + (z + 1) = 3 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x \cdot (y + 1) + (y + 1) = 2 \\ y \cdot (z + 1) + (z + 1) = 6 \\ x \cdot (z + 1) + (z + 1) = 3 \end{array}\right\} \Rightarrow \left.\begin{array}{l} (y + 1) \cdot (x + 1) = 2 \\ (z + 1) \cdot (y + 1) = 6 \\ (z + 1) \cdot (x + 1) = 3 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{matrix}\text{pomnožimo}\\ \text{jednadžbe}\end{matrix}\right] \Rightarrow \left.\begin{matrix}(y+1)\cdot(x+1)=2\\(z+1)\cdot(y+1)=6\\(z+1)\cdot(x+1)=3\end{matrix}\right\} \Rightarrow (x+1)^2\cdot(y+1)^2\cdot(z+1)^2=36 \Rightarrow$$

$$\Rightarrow ((x+1)\cdot(y+1)\cdot(z+1))^2=36 \Rightarrow ((x+1)\cdot(y+1)\cdot(z+1))^2=36 \; / \sqrt{\;} \Rightarrow$$

$$\Rightarrow (x+1)\cdot(y+1)\cdot(z+1)=\pm\sqrt{36} \Rightarrow (x+1)\cdot(y+1)\cdot(z+1)=\pm 6 \Rightarrow$$

$$\Rightarrow \left.\begin{matrix}(x+1)\cdot(y+1)\cdot(z+1)=6\\(x+1)\cdot(y+1)\cdot(z+1)=-6\end{matrix}\right\} \Rightarrow \left.\begin{matrix}(x+1)\cdot(y+1)\cdot(z+1)=1\cdot 2\cdot 3\\(x+1)\cdot(y+1)\cdot(z+1)=-1\cdot(-2)\cdot(-3)\end{matrix}\right\}.$$

**Prva jednadžba**

$$(x+1)\cdot(y+1)\cdot(z+1)=1\cdot 2\cdot 3 \Rightarrow \left.\begin{matrix}x+1=1\\y+1=2\\z+1=3\end{matrix}\right\} \Rightarrow \left.\begin{matrix}x=1-1\\y=2-1\\z=3-1\end{matrix}\right\} \Rightarrow \left.\begin{matrix}x=0\\y=1\\z=2\end{matrix}\right\}.$$

**Druga jednadžba**

$$(x+1)\cdot(y+1)\cdot(z+1)=-1\cdot(-2)\cdot(-3) \Rightarrow \left.\begin{matrix}x+1=-1\\y+1=-2\\z+1=-3\end{matrix}\right\} \Rightarrow \left.\begin{matrix}x=-1-1\\y=-2-1\\z=-3-1\end{matrix}\right\} \Rightarrow \left.\begin{matrix}x=-2\\y=-3\\z=-4\end{matrix}\right\}.$$

Rješenja sustava su:

$$(x,\ y,\ z)\in\{(0,\ 1,\ 2),\ (-2,\ -3,\ -4)\}.$$



**Vježba 164**

Odmor!

**Rezultat:** …

**Zadatak 165 (Danijel, gimnazija)**

Riješite sustav $\begin{cases}\dfrac{3}{2\cdot x-5}+x-3\cdot y+4=0\\ \dfrac{5}{4\cdot x-10}-2\cdot x+6\cdot y+\dfrac{1}{2}=0\end{cases}.$

**Rješenje 165**

Ponovimo!

$$a\cdot\frac{b}{c}=\frac{a\cdot b}{c}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left.\begin{matrix}\dfrac{3}{2\cdot x-5}+x-3\cdot y+4=0\\ \dfrac{5}{4\cdot x-10}-2\cdot x+6\cdot y+\dfrac{1}{2}=0\end{matrix}\right\} \Rightarrow \left.\begin{matrix}\dfrac{3}{2\cdot x-5}+x-3\cdot y+4=0\\ \dfrac{5}{2\cdot(2\cdot x-5)}-2\cdot(x-3\cdot y)+\dfrac{1}{2}=0\end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} \frac{3}{2\cdot x-5}+x-3\cdot y+4=0 \\ \frac{5}{2\cdot(2\cdot x-5)}-2\cdot(x-3\cdot y)+\frac{1}{2}=0 \;/\cdot 2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \frac{3}{2\cdot x-5}+x-3\cdot y+4=0 \\ \frac{5}{2\cdot x-5}-4\cdot(x-3\cdot y)+1=0 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \text{zamjena} \\ u=\frac{1}{2\cdot x-5} \\ v=x-3\cdot y \end{array}\right] \Rightarrow \left.\begin{array}{l} 3\cdot u+v+4=0 \\ 5\cdot u-4\cdot v+1=0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 3\cdot u+v=-4 \\ 5\cdot u-4\cdot v=-1 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} 3\cdot u+v=-4 \;/\cdot 4 \\ 5\cdot u-4\cdot v=-1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 12\cdot u+4\cdot v=-16 \\ 5\cdot u-4\cdot v=-1 \end{array}\right\} \Rightarrow 17\cdot u=-17 \Rightarrow 17\cdot u=-17 \;/:17 \Rightarrow u=-1.$$

Računamo v.

$$\left.\begin{array}{l} u=-1 \\ 3\cdot u+v=-4 \end{array}\right\} \Rightarrow 3\cdot(-1)+v=-4 \Rightarrow -3+v=-4 \Rightarrow v=-4+3 \Rightarrow v=-1.$$

**Vraćamo se zamjeni.**

$$\left.\begin{array}{l} u=\frac{1}{2\cdot x-5} \\ v=x-3\cdot y \end{array}\right\} \Rightarrow \left[\begin{array}{l} u=-1 \\ v=-1 \end{array}\right] \Rightarrow \left.\begin{array}{l} -1=\frac{1}{2\cdot x-5} \\ -1=x-3\cdot y \end{array}\right\} \Rightarrow \left.\begin{array}{l} -1=\frac{1}{2\cdot x-5} \;/\cdot(2\cdot x-5) \\ -1=x-3\cdot y \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} -2\cdot x+5=1 \\ -x+3\cdot y=1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} -2\cdot x=1-5 \\ -x+3\cdot y=1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} -2\cdot x=-4 \\ -x+3\cdot y=1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} -2\cdot x=-4 \;/:(-2) \\ -x+3\cdot y=1 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x=2 \\ -x+3\cdot y=1 \end{array}\right\} \Rightarrow -2+3\cdot y=1 \Rightarrow 3\cdot y=1+2 \Rightarrow 3\cdot y=3 \Rightarrow 3\cdot y=3 \;/:3 \Rightarrow y=1.$$

Rješenje sustava glasi:

$$(x,\ y)=(2,\ 1).$$



**Vježba 165**

Odmor!

**Rezultat:** …

**Zadatak 166 (Darko, maturant)**

Odredite y iz sustava jednadžba $\left\{\begin{array}{l} 3\cdot x+8\cdot y+12=0 \\ 4\cdot y^2=9\cdot x \end{array}\right.$.

**Rješenje 166**

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad a\cdot\frac{b}{c}=\frac{a\cdot b}{c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\left.\begin{array}{l}3\cdot x+8\cdot y+12=0\\4\cdot y^2=9\cdot x\end{array}\right\}\Rightarrow\left.\begin{array}{l}3\cdot x+8\cdot y+12=0\ /\cdot 3\\4\cdot y^2=9\cdot x\end{array}\right\}\Rightarrow\left.\begin{array}{l}9\cdot x+24\cdot y+36=0\\4\cdot y^2=9\cdot x\end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{metoda}\\\text{zamjene}\end{array}\right]\Rightarrow$$

$$\Rightarrow 4\cdot y^2+24\cdot y+36=0\Rightarrow 4\cdot y^2+24\cdot y+36=0\ /:4\Rightarrow y^2+6\cdot y+9=0\Rightarrow$$

$$\Rightarrow (y+3)^2=0\Rightarrow y+3=0\Rightarrow y=-3.$$

2.inačica

$$\left.\begin{array}{l}3\cdot x+8\cdot y+12=0\\4\cdot y^2=9\cdot x\end{array}\right\}\Rightarrow\left.\begin{array}{l}3\cdot x+8\cdot y+12=0\\4\cdot y^2=9\cdot x\ /:3\end{array}\right\}\Rightarrow\left.\begin{array}{l}3\cdot x+8\cdot y+12=0\\\frac{4}{3}\cdot y^2=3\cdot x\end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{metoda}\\\text{zamjene}\end{array}\right]\Rightarrow$$

$$\Rightarrow \frac{4}{3}\cdot y^2+8\cdot y+12=0\Rightarrow \frac{4}{3}\cdot y^2+8\cdot y+12=0\ /\cdot\frac{3}{4}\Rightarrow y^2+6\cdot y+9=0\Rightarrow$$

$$\Rightarrow (y+3)^2=0\Rightarrow y+3=0\Rightarrow y=-3.$$

3.inačica

$$\left.\begin{array}{l}3\cdot x+8\cdot y+12=0\\4\cdot y^2=9\cdot x\end{array}\right\}\Rightarrow\left.\begin{array}{l}3\cdot x=-8\cdot y-12\\4\cdot y^2=9\cdot x\end{array}\right\}\Rightarrow\left.\begin{array}{l}3\cdot x=-8\cdot y-12\ /:3\\4\cdot y^2=9\cdot x\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}x=-\frac{8}{3}\cdot y-4\\4\cdot y^2=9\cdot x\end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{metoda}\\\text{zamjene}\end{array}\right]\Rightarrow 4\cdot y^2=9\cdot\left(-\frac{8}{3}\cdot y-4\right)\Rightarrow 4\cdot y^2=-24\cdot y-36\Rightarrow$$

$$\Rightarrow 4\cdot y^2+24\cdot y+36=0\Rightarrow 4\cdot y^2+24\cdot y+36=0\ /:4\Rightarrow y^2+6\cdot y+9=0\Rightarrow$$

$$\Rightarrow (y+3)^2=0\Rightarrow y+3=0\Rightarrow y=-3.$$



**Vježba 166**

Odmor!

**Rezultat:** …

**Zadatak 167 (Zlatko, maturant)**

Koliko uređenih parova realnih brojeva zadovoljava ovaj sustav $\left\{\begin{array}{l}x+3\cdot|y|=1\\x+y=-3\end{array}\right.$?

**Rješenje 167**

Ponovimo!

Za realni broj x njegova je apsolutna vrijednost (modul) broj $|x|$ koji određujemo na ovaj način:

$$|x|=\left\{\begin{array}{l}x,\ x\geq 0\\-x,\ x<0.\end{array}\right.$$

Ako je broj x pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki x, $x \geq 0$, vrijedi $|x| = x$.

Ako je x negativan broj, njegova apsolutna vrijednost je suprotan broj – x koji je pozitivan. Za svaki x, $x < 0$, je $|x| = -x$.

$$\left.\begin{array}{l}x+3\cdot|y|=1\\x+y=-3\end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{metoda suprotnih}\\\text{koeficijenata}\end{array}\right]\Rightarrow\left.\begin{array}{l}x+3\cdot|y|=1\\x+y=-3\ /\cdot(-1)\end{array}\right\}\Rightarrow\left.\begin{array}{l}x+3\cdot|y|=1\\-x-y=3\end{array}\right\}\Rightarrow$$

$$\Rightarrow 3 \cdot |y| - y = 4.$$

**1.slučaj**

$y \geq 0 \Rightarrow |y| = y.$

$$3 \cdot |y| - y = 4 \Rightarrow 3 \cdot y - y = 4 \Rightarrow 2 \cdot y = 4 \Rightarrow 2 \cdot y = 4 \ /: 2 \Rightarrow y_1 = 2 \geq 0.$$

Računamo $x_1$.

$$\left.\begin{array}{l} y = 2 \\ x + y = -3 \end{array}\right\} \Rightarrow x + 2 = -3 \Rightarrow x = -3 - 2 \Rightarrow x_1 = -5.$$

Rješenje je uređeni par

$$(x_1, y_1) = (-5, 2).$$

**2.slučaj**

$y < 0 \Rightarrow |y| = -y.$

$$3 \cdot |y| - y = 4 \Rightarrow 3 \cdot (-y) - y = 4 \Rightarrow -3 \cdot y - y = 4 \Rightarrow -4 \cdot y = 4 \Rightarrow$$

$$\Rightarrow -4 \cdot y = 4 \ /: (-4) \Rightarrow y_2 = -1 < 0.$$

Računamo $x_2$.

$$\left.\begin{array}{l} y = -1 \\ x + y = -3 \end{array}\right\} \Rightarrow x - 1 = -3 \Rightarrow x = -3 + 1 \Rightarrow x_2 = -2.$$

Rješenje je uređeni par

$$(x_2, y_2) = (-2, -1).$$



Postoje dva uređena para.

**Vježba 167**

Odmor!

**Rezultat:** …

**Zadatak 168 (Medo, maturant)**

Ivan, Matija i Petar zajedno štede. Ivan je uštedio pet puta manje od Petra, a Petar je uštedio 425 kn više od Ivana i Matije zajedno. Ivan je uštedio pedeset kuna više od Matije. Koliko su kuna uštedjeli zajedno?

$A.\ 630\ kn \qquad B.\ 715\ kn \qquad C.\ 825\ kn \qquad D.\ 975\ kn$

**Rješenje 168**

Ponovimo!

Kako zapisati da je broj b n puta manji od broja a?

$$n \cdot b = a \quad , \quad b = \frac{1}{n} \cdot a \quad , \quad \frac{a}{b} = n.$$

Kako zapisati da je broj b za n veći od broja a?

$$b = a + n \quad , \quad b - n = a \quad , \quad b - a = n.$$

Neka je:

- i broj kuna koje je uštedio Ivan
- m broj kuna koje je uštedio Matija
- p broj kuna koje je uštedio Petar.

Ivan je uštedio pet puta manje od Petra. Matematički to zapisujemo u obliku sljedeće jednadžbe:

$$5 \cdot i = p.$$

Petar je uštedio 425 kn više od Ivana i Matije zajedno. Matematički zapis glasi:

$$p = (i + m) + 425 \Rightarrow p = i + m + 425.$$

Ivan je uštedio pedeset kuna više od Matije. Matematički to zapisujemo ovako:

$$i = m + 50.$$

Dobili smo sustav! Riješimo ga!

$$\left.\begin{aligned} 5 \cdot i &= p \\ p &= i + m + 425 \\ i &= m + 50 \end{aligned}\right\} \Rightarrow \left[\begin{array}{l} \text{iz prve jednadžbe } p \\ \text{uvrstimo u drugu} \end{array}\right] \Rightarrow \left.\begin{aligned} 5 \cdot i &= i + m + 425 \\ i - 50 &= m \end{aligned}\right\} \Rightarrow \left[\begin{array}{l} \text{iz druge jednadžbe } m \\ \text{uvrstimo u prvu} \end{array}\right] \Rightarrow$$

$$\Rightarrow 5 \cdot i = i + i - 50 + 425 \Rightarrow 5 \cdot i - i - i = -50 + 425 \Rightarrow 3 \cdot i = 375 \Rightarrow 3 \cdot i = 375 \;/ : 3 \Rightarrow i = 125.$$

Računamo p.

$$\left.\begin{aligned} i &= 125 \\ p &= 5 \cdot i \end{aligned}\right\} \Rightarrow p = 5 \cdot 125 \Rightarrow p = 625.$$

Računamo m.

$$\left.\begin{aligned} i &= 125 \\ i &= m + 50 \end{aligned}\right\} \Rightarrow m + 50 = 125 \Rightarrow m = 125 - 50 \Rightarrow m = 75.$$

Oni su zajedno uštedjeli 825 kn.

$$125 \; kn + 75 \; kn + 625 \; kn = 825 \; kn.$$

Odgovor je pod C.

**Vježba 168**

Odmor!

**Rezultat:** …



**Zadatak 169 (Tomislav, gimnazija)**

Riješite u skupu cijelih brojeva sustav $\begin{cases} x^2 - y^2 - z^2 = 1 \\ y + z - x = 3 \end{cases}.$

**Rješenje 169**

Ponovimo!

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2 \cdot a \cdot b + 2 \cdot a \cdot c + 2 \cdot b \cdot c \quad , \quad \frac{a + b}{n} = \frac{a}{n} + \frac{b}{n} \quad , \quad \frac{n}{1} = n.$$

Cijeli brojevi jesu brojevi:

$$\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots$$

Oni čine skup cijelih brojeva koji označavamo slovom Z, a zapisujemo kao

$$Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} \qquad \text{ili} \qquad Z = \{0, 1, -1, 2, -2, 3, -3, \ldots\}.$$

Za prirodni broj a kažemo da je djeljiv s prirodnim brojem b ako postoji prirodan broj q tako da vrijedi

$$a = b \cdot q.$$

Broj q zovemo količnikom brojeva a i b i pišemo

$$\frac{a}{b} = q \quad \text{ili} \quad a : b = q.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \;,\; n \ne 0 \;,\; n \ne 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

$$\left.\begin{array}{l}x^2-y^2-z^2=1\\ y+z-x=3\end{array}\right\} \Rightarrow \left.\begin{array}{l}x^2-y^2-z^2=1\\ -x=3-y-z\end{array}\right\} \Rightarrow \left.\begin{array}{l}x^2-y^2-z^2=1\\ -x=3-y-z \ /\cdot(-1)\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}x^2-y^2-z^2=1\\ x=y+z-3\end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{metoda}\\ \text{zamjene}\end{array}\right] \Rightarrow (y+z-3)^2-y^2-z^2=1 \Rightarrow$$

$$\Rightarrow y^2+z^2+9+2\cdot y\cdot z-6\cdot y-6\cdot z-y^2-z^2=1 \Rightarrow$$

$$\Rightarrow y^2+z^2+9+2\cdot y\cdot z-6\cdot y-6\cdot z-y^2-z^2=1 \Rightarrow 9+2\cdot y\cdot z-6\cdot y-6\cdot z=1 \Rightarrow$$

$$\Rightarrow 2\cdot y\cdot z-6\cdot y-6\cdot z=1-9 \Rightarrow 2\cdot y\cdot z-6\cdot y-6\cdot z=-8 \Rightarrow 2\cdot y\cdot z-6\cdot y-6\cdot z=-8 \ /:2 \Rightarrow$$

$$\Rightarrow y\cdot z-3\cdot y-3\cdot z=-4 \Rightarrow y\cdot(z-3)-3\cdot z=-4 \Rightarrow y\cdot(z-3)=3\cdot z-4 \Rightarrow$$

$$\Rightarrow y\cdot(z-3)=3\cdot z-4 \ /\cdot\frac{1}{z-3} \Rightarrow y=\frac{3\cdot z-4}{z-3} \Rightarrow y=\frac{3\cdot z-9+5}{z-3} \Rightarrow y=\frac{3\cdot z-9}{z-3}+\frac{5}{z-3} \Rightarrow$$

$$\Rightarrow y=\frac{3\cdot(z-3)}{z-3}+\frac{5}{z-3} \Rightarrow y=\frac{3\cdot(z-3)}{z-3}+\frac{5}{z-3} \Rightarrow y=3+\frac{5}{z-3}.$$

Da bi y bio cijeli broj mora izraz $\frac{5}{z-3}$ biti cijeli broj. To je moguće ako je

$$\left.\begin{array}{l}z-3=-5\\ z-3=-1\\ z-3=1\\ z-3=5\end{array}\right\} \Rightarrow \left.\begin{array}{l}z=-5+3\\ z=-1+3\\ z=1+3\\ z=5+3\end{array}\right\} \Rightarrow \left.\begin{array}{l}z_1=-2\\ z_2=2\\ z_3=4\\ z_4=8\end{array}\right\}.$$



Računamo $y_1$, $y_2$, $y_3$ i $y_4$.

- $\left.\begin{array}{l}z=-2\\ y=\frac{3\cdot z-4}{z-3}\end{array}\right\} \Rightarrow y=\frac{3\cdot(-2)-4}{-2-3} \Rightarrow y=\frac{-6-4}{-5} \Rightarrow y=\frac{-10}{-5} \Rightarrow y=\frac{-10}{-5} \Rightarrow y_1=2$
- $\left.\begin{array}{l}z=2\\ y=\frac{3\cdot z-4}{z-3}\end{array}\right\} \Rightarrow y=\frac{3\cdot 2-4}{2-3} \Rightarrow y=\frac{6-4}{-1} \Rightarrow y=\frac{2}{-1} \Rightarrow y_2=-2$
- $\left.\begin{array}{l}z=4\\ y=\frac{3\cdot z-4}{z-3}\end{array}\right\} \Rightarrow y=\frac{3\cdot 4-4}{4-3} \Rightarrow y=\frac{12-4}{1} \Rightarrow y=\frac{8}{1} \Rightarrow y_3=8$
- $\left.\begin{array}{l}z=8\\ y=\frac{3\cdot z-4}{z-3}\end{array}\right\} \Rightarrow y=\frac{3\cdot 8-4}{8-3} \Rightarrow y=\frac{24-4}{5} \Rightarrow y=\frac{20}{5} \Rightarrow y=\frac{20}{5} \Rightarrow y_4=4.$

Sada računamo $x_1$, $x_2$, $x_3$ i $x_4$.

- $\left.\begin{array}{l}y=2 \ , \ z=-2\\ x=y+z-3\end{array}\right\} \Rightarrow x=2-2-3 \Rightarrow x=2-2-3 \Rightarrow x_1=-3$

- $\left.\begin{matrix} y=-2 \ , \ z=2 \\ x=y+z-3 \end{matrix}\right\} \Rightarrow x=-2+2-3 \Rightarrow x=-2+2-3 \Rightarrow x_2=-3$
- $\left.\begin{matrix} y=8 \ , \ z=4 \\ x=y+z-3 \end{matrix}\right\} \Rightarrow x=8+4-3 \Rightarrow x_3=9$
- $\left.\begin{matrix} y=4 \ , \ z=8 \\ x=y+z-3 \end{matrix}\right\} \Rightarrow x=4+8-3 \Rightarrow x_4=9.$

Rješenja sustava su:

$$(x_1, y_1, z_1)=(-3, 2, -2) \quad , \quad (x_2, y_2, z_2)=(-3, -2, 2)$$
$$(x_3, y_3, z_3)=(9, 8, 4) \quad , \quad (x_4, y_4, z_4)=(9, 4, 8).$$

**Vježba 169**

Odmor!

**Rezultat:** …

**Zadatak 170 (Marin, maturant)**

Nejednadžba $\frac{x-1}{x+2}<0$ zadovoljena je za

$A.\ x \in \langle -\infty, 1 \rangle \qquad B.\ x \in \langle -2, +\infty \rangle \qquad C.\ x \in \langle -\infty, -2 \rangle \qquad D.\ x \in \langle -2, 1 \rangle$

**Rješenje 170**

Ponovimo!

$$\frac{a}{b}<0 \Rightarrow \left.\begin{matrix} a>0 \ , \ b<0 \\ a<0 \ , \ b>0 \end{matrix}\right\} \quad , \quad x>a \Rightarrow x \in \langle a, +\infty \rangle \quad , \quad x<a \Rightarrow x \in \langle -\infty, a \rangle.$$

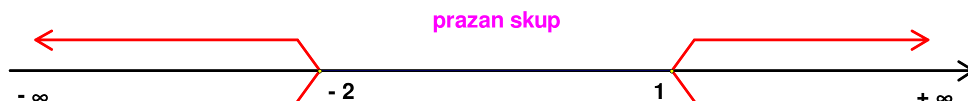
Skup zadajemo nabrajanjem njegovih elemenata ili opisom karakterističnih svojstava koja posjeduju njegovi elementi. Presjek skupova A i B je skup koji sadrži sve elemente koji se nalaze i u skupu A i u skupu B. Označavamo ga $A \cap B$. Znak $\varnothing$ simbol je za prazan skup ili skup bez elemenata.



**Prvi slučaj**

**Razlomak je negativan, ako mu je brojnik pozitivan, a nazivnik negativan.**

$$\frac{x-1}{x+2}<0 \Rightarrow \left.\begin{matrix} x-1>0 \\ x+2<0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x>1 \\ x<-2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x \in \langle 1, +\infty \rangle \\ x \in \langle -\infty, -2 \rangle \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{presjek} \\ \text{skupova} \end{matrix}\right] \Rightarrow \text{prazan skup}, \varnothing.$$

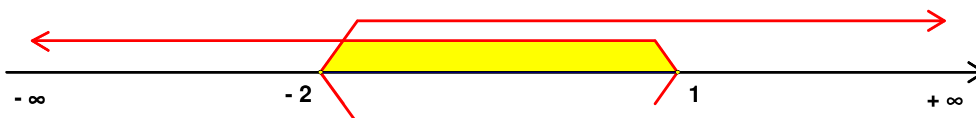



Odgovor je pod D.

**Drugi slučaj**

**Razlomak je negativan, ako mu je brojnik negativan, a nazivnik pozitivan.**

$$\frac{x-1}{x+2}<0 \Rightarrow \left.\begin{matrix} x-1<0 \\ x+2>0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x<1 \\ x>-2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x \in \langle -\infty, 1 \rangle \\ x \in \langle -2, +\infty \rangle \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{presjek} \\ \text{skupova} \end{matrix}\right] \Rightarrow x \in \langle -2, 1 \rangle.$$




Odgovor je pod D.

**Vježba 170**

Odmor!

**Rezultat:** …